From : Chapra & Canale 1 - 5 - 2021

Quadratic Interpolation $\longrightarrow$ it provide better estimation

Quadratic polynomial :-

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$\Downarrow$$

Expand

$$\Downarrow$$

$$f(x) = b_0 + b_1 x - b_1 x_0 + b_2 x^2 + b_2 x_0 \cdot x_1 - b_2 x \cdot x_0 - b_2 x \cdot x_1$$

collecting terms,

$$f(x) = a_0 + a_1 x + a_2 x^2$$

where,

$$a_0 = b_0 - b_1 x_0 + b_2 \cdot x_0 \cdot x_1$$

$$a_1 = b_1 - b_2 x_0 - b_2 \cdot x_1$$

$$a_2 = b_1$$

Say we have,

$x_0 = 1$ $\quad f(x_0) = 0$

$x_1 = 4$ $\quad f(x_1) = 1.386294$

$x_2 = 6$ $\quad f(x_2) = 1.791759$

$$b_0 = f(x_0)$$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$b_0 = 0$$

$$b_1 = \frac{1.386294 - 0}{4 - 1} = 0.4620981$$

$$b_2 = \frac{\dfrac{1.791759 - 1.386294}{6 - 4} - 0.4620981}{6 - 1} = -0.0518731$$

Thus,

$$f(x) = 0 + 0.4620981(x - 0) - 0.0518731(x - 0)(x - 4)$$

$$f(x) = 0.4620981x - 0.0518731x(x - 4)$$

You can use this function to interpolate results, preferedly within the range of known data, that is $1 < x < 6$

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