

Finding Arc Length by Integration

Suppose we have the suspension bridge as shown on the right,

$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

The appropriate integration formula is :-

$$\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2})$$

Since  $y = 0.0625x^2 - 5x + 100$

then  $u = \frac{dy}{dx} = 0.125x - 5$ , and  $\frac{du}{dx} = 0.125$

Thus,  $\overset{80}{\curvearrowright}$  8 compensate for 0.125  $du = 0.125 dx$

$$s = 8 \int_0^{80} \sqrt{1 + (0.125x - 5)^2} [0.125 dx]$$

$$= 8 \left[ \frac{0.125x - 5}{2} \sqrt{1 + (0.125x - 5)^2} + \frac{1}{2} \ln(0.125x - 5 + \sqrt{1 + (0.125x - 5)^2}) \right]_0^{80}$$

$$= 8(2.5 \times 5.099 + 1.156) - 8(-2.5 \times 5.099 - 1.156)$$

$$s = 222.46 \text{ m} \neq$$

